A Summary of the UGC minor Research Project

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entitled "SOME PROBLEMS IN GRAPH THEORY WITH SPECIAL REFERENCE TO FUZZY INTERSECTION GRAPHS"

The branch of mathematics called "Graph Theory" has shown to be more useful in the modelling the essential features of systems with a finite number of components. Intersection graphs and, in particular, interval graphs have been used extensively in mathematical modelling. Roberts cites applications in archaeology, developmental psychology, mathematical sociology, organization theory, and ecological modelling. These disciplines all have components that are ambiguously defined, require subjective evaluation, or are satisfied to differing degrees; thus these areas can benefit from an application of fuzzy methods.

Another focus of the present study is on various types of fuzzy intersection graph and their polysemic intersection representations. The idea of polysemy has applications not only in graph theory, but also in other applied areas like computer communication and scheduling. Polysemy expresses the interactions among various classes of abstract objects on a common ground set. In graph polysemy, a single set of objects is seen under multiple interpretations to represent several graphs simultaneously. Consequently, it ensures the existence of a single representation for multiple posets and different graphs.

After development of fuzzy graph theory by Rosenfeld, the fuzzy graph theory is increased with a large number of branches. In this study, an attempt is made to present the generalization which is afforded by introducing the concept of fuzzy sets into the theory of graphs, i.e. a generalization of intersection graphs to fuzzy intersection graphs.

Given a finite family of fuzzy sets $\mathcal{E}$**,** McAllister defines two structures which together are called a fuzzy intersection graph. The first is essentially a fuzzy hypergraph with edge set consisting of all nonempty intersections of two distinct members of $\mathcal{E}$. The second is a fuzzy graph with a crisp vertex set (essentially $\mathcal{E}$) where the edge strength of a pair (α, β) given by a "measure of fuzzyness of $\alpha \wedge \beta$**".** Both structures are represented by incidence matrices with the α, β entry being the membership function a $\alpha \wedge \beta$ or the edge strength of $\{\alpha, \beta\}$, respectively. McAllister's main concern was to explore when linear algebra methods could be used to study eigenvalues, stability, or other properties of these matrices. We note that neither of McAllister's structures agree with the usual definition of intersection graphs when applied to families of crisp sets.

We take a different approach in defining the fuzzy intersection graph of a finite family of fuzzy sets. Our structure is a fuzzy graph where the fuzzy vertex and fuzzy edge sets are based on the max and min operators.

DEFINITION : Let $\mathcal{F} = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ be a finite family of fuzzy sets on a set X and consider $\mathcal{F}$ as a crisp vertex set. The fuzzy *intersection graph* of $\mathcal{F}$ is the fuzzy graph $Int(\mathcal{F}) = (\sigma, \mu)$ where $\sigma: \mathcal{F} \to [0,1]$ defined by $\sigma(\alpha_1) = h(\alpha_1)$ and $\mu: \mathcal{F} \times \mathcal{F} \to [0,1]$ is defined by $\mu(\alpha_i, \alpha_j) = \begin{cases} h\big(\alpha_i \wedge \alpha_j\big) & if\ i \neq j \\ 0 & if\ i = j \end{cases}$

An edge $\big(\alpha_i, \alpha_j\big)$ has zero strength if and only if $\alpha_i \wedge \alpha_j$ is the zero function (empty intersection) or $i \neq j$ (no loops).

The study also presents a theorem analogous to that in the case of crisp sets:

THEOREM **:** Let $\mathcal{G} = (\sigma, \mu)$ be a fuzzy graph without loops. Then there exists a family of fuzzy sets $\mathcal{F}$ where $\mathcal{G} = Int(\mathcal{F})$.
